# **Linear Discriminant Function of Classification problem**

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# **Linear Discriminant Function of Classification problem**

According to Bayes classification theorem, the posterior probability P(yq|x) can be expressed in terms of prior probability P(yq) and the conditional probability density function p(x| yq). According to Gopal (2019), this can be expressed in the form of Bayes rule as,

P(yq|x ) = ( p(x| yq) P(yq) ) / Σq=1,2 p(x| yq) P(yq) ) 🡪 **Eqn 1**

Now given p(x| yq) = (1/( 2π)n/2 |Σ| ½ ) exp(-0.5 (x-μq)T Σ-1(x-μq) ); q=1,2 🡪 **Eqn 2**

Let’s substitute the value from Eqn 2 in Eqn 1, we can write Eqn 1 as below,

P(yq|x ) = exp(-0.5 (x-μq)T Σ-1(x-μq) ) P(yq) / ( ( 2π)n/2 |Σ| ½ Σq=1,2 p(x| yq) P(yq) )🡪 **Eqn 3**

Apply Natural logarithm on both side of the Eqn 3.

ln(P(yq|x ))= -0.5 ( (x-μq)T Σ-1(x-μq) ) + ln (P(yq))

= -0.5 (xT Σ-1x -2 xT Σ-1 μq + μqT Σ-1 μq ) + ln (P(yq))

In the above equation the xT Σ-1x is a constant. Hence the above equation can be simplified as,

gq(x) = μqTΣ-1x – 0.5μqT Σ-1μq + ln P(yq) ; q=1,2 🡪 **Eqn 4**

The above equation is the linear discriminant function of a Bayesian classifier

Now, according to Gopal (2019), in case of binary classification two linear discriminants g1(x) and g2(x) can be combined to form a single discriminant function,

g(x) = g1(x) – g2(x) = 0 🡪 **Eqn 5**

Let’s substitute the values of g1(x) and g2(x) from Eqn 4 in Eqn 5.

g(x) = μ1TΣ-1x – 0.5μ1T Σ-1μ1 + ln P(y1) – (μ2TΣ-1x – 0.5μ2T Σ-1μ2 + ln P(y2))

* μ1TΣ-1x – 0.5μ1T Σ-1μ1 + ln P(y1) - μ2TΣ-1x + 0.5μ2T Σ-1μ2 - ln P(y2)
* (μ1T - μ2T) Σ-1x -0.5 (μ1T Σ-1μ1 - μ2T Σ-1μ2) + ln P(y1) - ln P(y2)
* (μ1T - μ2T) Σ-1x -0.5 (μ1T Σ-1μ1 - μ2T Σ-1μ2) + ln ( P(y1) / P(y2) )

Hence we can prove that,

g(x) = (μ1T - μ2T) Σ-1x -0.5 (μ1T Σ-1μ1 - μ2T Σ-1μ2) + ln ( P(y1) / P(y2) ) 🡪 **Eqn 6**

# **References**

Gopal, M. (2019). *Applied machine learning*. McGraw-Hill Education.

Hoa, N. (2015, December 29). Discriminant analysis & Naive Bayes. http://jennguyen1.github.io/nhuyhoa/statistics/Discriminant-Analysis-Naive-Bayes.html.